NAME:

Math 150 Practice Exam 3.2

Instructions: WRITE YOUR NAME CLEARLY. Do as many problems as you can for a maximal score of 100. SHOW YOUR WORK!

1. Find a simple expression for $\int \frac{4x^4 - 6x^2}{x} dx$ [10 pts]

2. Find a simple expression for $\int \frac{\sin\theta - 1}{\cos^2\theta} d\theta$ [10 pts]

3. Use geometry to evaluate $\int_{-1}^{3} \sqrt{4 - (x + 1)^2}\,dx$ [10 pts]

4. Use Riemann sums to evaluate $\int_{3}^{7} (4x + 6)\,dx$ [10 pts]

5. Compute $\lim_{n \to \infty} \frac{\pi}{2n} \left(\sin\left(\pi - 1\frac{\pi}{2n} \right) + \sin\left(\pi - 2\frac{\pi}{2n} \right) + \cdots + \sin\left(\pi - n\frac{\pi}{2n} \right) \right)$

[10 pts]

6. Find $\frac{d}{dx} \int_x^{x^2} \sin t^2 \, dt$

[10 pts]

7. Compute $\int_{-1}^{1} \sin(\pi x^3)\,dx$. Be sure to justify your answer. [10 pts]

8. Calculate $\int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin^2 x}\,dx$ [10 pts]

9. Find a simple expression for $\int \frac{x}{\sqrt{4-9x^2}} dx$ [10 pts]

10. Calculate $\lim_{h \to 0} \frac{1}{h} \int_0^h f(x)dx$, where $f(x) = \begin{cases} \frac{\sin 2x}{x} & if\ x \neq 0 \\ 5 & if\ x = 0 \end{cases}$

[10 pts]

Extra-Credit

11. Let $F(x) = \int_0^x t^2\, dt$ and $G(x) = \int_0^x x^2\, dx$. Is there any difference between the two functions? Justify your answer. [10 pts]

12. Let $G(x) = \int_x^{\int_0^x v\, dv} \cos(t^2)dt$. Find $G'(x)$ [10 pts]

13. Show that $\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$ [10 pts]

14. Suppose that f is an even function with $\int_0^8 f(x)dx = 9$. Evaluate
$\int_{-2}^{2} x^2 f(x^3)dx.$ [10 pts]